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# Cournot Competition and Access Pricing in Regulated Industries

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## 1 Introduction

We analyse the equilibrium of an economy where one of the producers in the final market is the monopolistic supplier of input needed by both its owner and competitors, and monopoly pricing behavior of the firm is regulated. Our particular interest is in the pricing of monopoly input which is supposed to be regulated by the government. Typical examples of the model include electricity transmissions, access to local telecommunications services and network provisions in other utility industries.

Two major problems we address in this paper are:

(i) How high should the access price for the downstream firms be compared with the marginal cost of the upstream supplier ?

(ii) Is free entry to the downstream market desirable ?

With regard to the first problem, we examine Baumol=Willig efficient component pricing rule (ECPR) which states that the access charge should be set roughly by the formula

Optimal access charge = direct cost of providing access + opportunity cost of  
providing access

Here, the opportunity cost is defined to be the reduction in incumbent's (monopolist's) profit caused by the provision of the service.

Pioneering studies are made by Laffont and Tirole(1993, 2000), Vickers(1995) and Armstrong, Doyle and Vickers(1996) and Economides and White (1995) among others, and more exact formulae are derived with additional features such as the incentive constraint and the monopoly element. When the budget constraint of the incumbent firm is an issue, their results are related to the classic Ramsey=Boiteux markup ratio formula but the analysis of optimal access charge is fairly complicated even in the case where the entrants act as price takers.

We study the case where entrants act as Cournot oligopolists as well as the case where they behave as price takers. We do not assume, to begin with, that profits of downstream firms are zero as most previous studies did. In fact, one of our chief objectives is to examine whether or not free entry (zero profit condition) is desirable from the welfare viewpoint.

As answers to the above problems, we give sufficient conditions under which the optimal access price is higher than the marginal cost (Propositions 2, 3 and 4) and show that entry to the downstream industry tends to be excessive (Proposition 6). Some specific studies are made in the case of homogeneous product and in the case of perfectly competitive downstream market. We will also discuss the implications of classical optimal taxation literature to the current problem (Proposition 5). It is hoped that our analysis helps to unify and relate previous results scattered in the literature of different fields.

## 2 Model

There are two kind of final goods in the economy. Good  $Y$  is produced by the monopolist  $M$  which is regulated by the government. Good  $X$  is produced by a small number of firms which behave as Cournot oligopolists (or as price takers as a special case).

Formally this does not exclude the possibility that goods  $X$  and  $Y$  are homogeneous and we will be concerned with this special case occasionally. The monopolist owns the input(infrastructure) which is necessary for the production of goods  $X$  and  $Y$ .

The total utility of the representative consumer who consumes  $X$  units

of good  $X$  and  $Y$  units of good  $Y$  and works  $L$  hours are expressed as

$$W = U(X, Y) + (L_0 - L), \quad (2.1)$$

where  $L_0$  is a fixed number indicating the initial endowment of labour. We assume that  $U$  is twice continuously differentiable and set

$$P(X, Y) = U_X(X, Y) \quad (2.2)$$

and

$$Q(X, Y) = U_Y(X, Y), \quad (2.3)$$

where the subscripts denote the partial derivatives with respect to the designated variables. We remark that  $P_Y(X, Y) = Q_X(X, Y)$  by continuous differentiability of  $U$ , and assume that  $U$  is increasing and concave ( $W$  is quasi-concave). These assumptions may be expressed as :

A1

$$\begin{aligned} P(X, Y) &> 0, \quad Q(X, Y) > 0 \\ P_X(X, Y) &< 0, \quad Q_Y(X, Y) < 0 \end{aligned}$$

and

$$P_X(X, Y) \cdot Q_Y(X, Y) - P_Y(X, Y) \cdot Q_X(X, Y) > 0.$$

We also make assumption

A2

$$\begin{aligned} P(X, Y) + P_X(X, Y)X &> 0 \\ Q(X, Y) + Q_Y(X, Y)Y &> 0 \end{aligned}$$

and

$$\begin{aligned} 2P_X(X, Y) + P_{XX}(X, Y) \cdot X &< 0 \\ 2Q_Y(X, Y) + Q_{YY}(X, Y) \cdot Y &< 0. \end{aligned}$$

This means that the marginal revenue function of each industry, if monopolized, is positive and decreasing with respect to its own production.

Let  $x$  be the output level and  $c(x)$  be the corresponding cost of the typical downstream firm. The aggregate output of these firms is equal to the demand for the output  $X$ . We assume

A3

$$c'(x) \geq 0, \quad c''(x) \geq 0.$$

The last condition will be relaxed in some cases.

Let  $C(X, Y)$  be the cost function of the upstream firm (monopolist) to supply the service in the amount  $X$  to the downstream firms and to produce its monopolistic product in the amount  $Y$ . We assume

A4

$C(X, Y)$  is increasing in  $X$  and  $Y$  and convex in  $(X, Y)$ .

The convexity assumption will be relaxed in some context in the following discussion.

### 3 Cournot Competition in the Downstream Market

Let  $a$  denote the access price of the bottleneck input (infrastructure), the profit of downstream firm(entrant)  $i$  corresponding to the output pair  $(X, Y)$  is expressed as

$$\pi_i = (P(X, Y) - a)x_i - c(x_i). \quad (3.1)$$

Given  $Y$  and  $a$ , and assuming the Cournot behavior and the interior solution, the first order conditions for profit maximization are expressed as

$$P(X, Y) + P_X(X, Y) \cdot x_i = a + c'(x_i), \quad \text{all } i. \quad (3.2)$$

From (3.1) and (3.2) we note the following:

#### Remarks

- (i)  $P \geq a + C(x)/x$  if  $\pi_i$  is positive
- (ii)  $P \geq a + c'$  if  $P_X$  is negative

Now letting  $n$  denote the number of the downstream firms and assuming the symmetric solution ( $x = x_i$  for all  $i$ ), we have

$$X = nx \quad (3.3)$$

and

$$nP(X, Y) + P_X(X, Y) \cdot X = n(a + c'(x)). \quad (3.4)$$

We may rewrite (3.4) as

$$a = P(X, Y) + P_X(X, Y) \cdot x - c'(X/n). \quad (3.5)$$

Thus there exists a unique access price  $a$  that is compatible with the output pair  $(X, Y)$ . It turns out more convenient to interpret the first order condition in this way, rather than in the other way round, i. e., given  $Y$  and  $a$ , (3.4) determines industry output  $X$ . Of course these two interpretations are equivalent since the right hand side of (3.5) is a monotone function of  $X$  under our assumptions. In fact, we have

$$\frac{\partial a}{\partial X} = ((n+1)P_X + P_{XX} \cdot X - c''(x))/n. \quad (3.6)$$

We also have

$$\frac{\partial a}{\partial Y} = P_Y + P_{XY} \cdot x \quad (3.7)$$

Thus  $\partial a / \partial X < 0$  if good  $X$  and good  $Y$  are strategic substitutes in consumption in the sense that an increase in  $Y$  decreases the marginal revenue  $P + P_X X$  of  $X$ .

If the downstream industry is competitive, (3.5) reduces to

$$a = P(X, Y) - c'(X/n) \quad (3.8)$$

and this implies that

$$\frac{\partial a}{\partial X} = P_X - c''(x)/n. \quad (3.9)$$

$$\frac{\partial a}{\partial Y} = P_Y. \quad (3.10)$$

The profit of the monopolist corresponding to the output pair  $(X, Y)$  is given by

$$\Pi(X, Y) = a(X, Y)X + Q(X, Y) \cdot Y - C(X, Y)$$

where  $a = a(X, Y)$  is defined in (3.5). The first term represents the income from the access charge of the network, the second term, the revenue from the sale of the product, while the third term is the total cost of the supply of the products  $X$  and  $Y$ .

For future reference, we record the derivatives of the profit function with respect to  $X$  and  $Y$  :

$$\Pi_X = a(X, Y) + a_X(X, Y) \cdot X + Q_X(X, Y) \cdot Y - C_X(X, Y) \quad (3.11)$$

$$\Pi_Y = a_Y(X, Y) \cdot X + Q(X, Y) + Q_Y(X, Y) \cdot Y - C_Y(X, Y). \quad (3.12)$$

In the above expressions

$$a_X(X, Y) \cdot X = (n+1)P_X \cdot X/n + P_{XX} \cdot X^2/n - c'' \cdot x \quad (3.13)$$

and

$$a_Y(X, Y) \cdot X = P_Y X + P_{XY} X^2/n. \quad (3.14)$$

## 4 Cournot Competition in the Product Markets and the Optimal Access Price

With regard to the behavior of the incumbent (the monopolistic supplier of the input) and the regulatory objective of the government, we consider two hypothetical situations. In this section we assume that the incumbent competes with entrants in the product markets as a Cournot oligopolist taking the access price  $a$  as given and using  $Y$  as its strategic variable. This assumption is made by Economides and White (1995) among others.

Under this market condition, access price  $a$  is set by the government so as to maximize the social welfare (total surplus) defined by

$$W(X, Y) = U(X, Y) - C(X, Y) - nc(X/n) , \quad (4.1)$$

In the next section, we consider the problem of maximizing the social welfare subject to an incentive constraint (budget constraint) of the incumbent assuming Cournot behaviors of the entrants.

When the access price  $a$  is fixed, the profit of the incumbent is written as

$$\Pi(X, Y) = (a - Q(X, Y))Y - C(X, Y) , \quad (4.2)$$

Hence the maximization of the profit given  $X$  (Cournot competition assumption) yields

$$Q(X, Y) + Q_Y(X, Y)Y = C_Y(X, Y) \quad (4.3)$$

On the other hand, corresponding to each output pair  $(X, Y)$ , there exists the unique access price

$$a(X, Y) = P(X, Y) + P_X(X, Y)X/n - c'(X/n) \quad (4.4)$$

which is consistent with the Cournot equilibrium condition of the entrants' market (see the discussion following (3,5)).

We now introduce the marginal profit function

$$G(X, Y) = Q(X, Y) + Q_Y(X, Y)Y - C_Y(X, Y) \quad (4.5)$$

of the incumbent with respect to its controlled variable  $Y$ . Thus  $G(X, Y) = \Pi_Y(X, Y)$ .

Differentiating with respect to  $X$  and  $Y$ , we have

$$G_X(X, Y) = Q_X(X, Y) + Q_{YX}(X, Y)Y - C_{YX}(X, Y) \quad (4.6)$$



$$G_Y(X, Y) = 2Q_Y(X, Y) + Q_{YY}(X, Y)Y - C_{YY}(X, Y) . \quad (4.7)$$

Now the government's problem is to choose  $a = a(X, Y)$ , so as to maximize the welfare function (4.1) subject to Cournot equilibrium conditions (4.3) and (4.4). Since  $a$  is a given function of  $(X, Y)$ , we need only choose  $(X, Y)$  optimally under constraint (4.3) which is conveniently written as  $G(X, Y) = 0$

Writing the Lagrangian of the problem as

$$\mathcal{L} = U(X, Y) - C(X, Y) - nc(X/n) + \lambda G(X, Y) \quad (4.8)$$

we obtain the first order conditions for the maximization as

$$P - C_X - c' = -\lambda G_X \quad (4.9)$$

and

$$Q - C_Y = -\lambda G_Y \quad (4.10)$$

Hence eliminating  $\lambda$  and using (4.3) we derive

$$P - C_X - c' = -Q_Y Y G_X / G_Y \quad (4.11)$$

Also, using (4.4) and (4.11) we have

$$\begin{aligned} a &= P + P_X \cdot x - c' \quad (4.12) \\ &= (P - C_X - c') + (P_X \cdot x + C_X) \\ &= -Q_Y Y G_X / G_Y + P_X \cdot x + C_X. \end{aligned}$$

We call that good  $X$  and good  $Y$  are strategic substitutes in production if  $\Pi_{XY} < 0$  (i.e.  $G_X < 0$ ) and strategic complements if  $\Pi_{XY} > 0$  (i.e.  $G_X > 0$ ).  $X$  and  $Y$  are independent goods if  $\Pi_{XY} = 0$  (i.e.  $G_X = 0$ ).

We note that  $G_Y < 0$  from our assumptions A2 and A4. Hence, in view of (4.11), we are ready to state our basic result on the relationship between the price of good  $X$  and its direct and indirect costs:

Proposition 1

$P > C_X + c'$  or  $P < C_X + c'$  depending on whether goods  $X$  and  $Y$  are strategic substitutes or strategic complements.

From (4.12) we can also infer the relationship between the access price and its indirect cost:

Proposition 2

$a < C_X$  if goods  $X$  and good  $Y$  are strategic complements. Also, if the downstream market is competitive,  $a < C_X$  or  $a > C_X$  depending on whether goods  $X$  and  $Y$  are strategic substitutes or complements.

## 5 Welfare Maximization Under the Revenue Constraint of the Monopolist

We next introduce the revenue (incentive) constraint and consider the problem of maximizing the welfare function

$$W = U(X, Y) - C(X, Y) - nc(X/n). \quad (5.1)$$

See, Laffont and Tirole (1983 and 2000) and Armstrong, Doyle and Vickers (1996) among others.

Under the Cournot equilibrium conditions in the entrants' market, the constraint may be written as

$$\Pi(X, Y) = a(X, Y)X + Q(X, Y) \cdot Y - C(X, Y) \geq 0 \quad (5.2)$$

where  $a(X, Y)$  is defined by (4.4).

We set the Lagrangian of the constrained maximization problem as

$$\mathcal{L} = U(X, Y) - C(X, Y) - nc(X/n) + \lambda \Pi(X, Y) \quad (5.3)$$

and differentiating with respect to  $X$  and  $Y$ , obtain

$$P(X, Y) - C_X(X, Y) - c'(x) + \lambda \Pi_X(X, Y) = 0 \quad (5.4)$$

and

$$Q(X, Y) - C_Y(X, Y) + \lambda \Pi_Y(X, Y) = 0 \quad (5.5)$$

(5.5) may be expressed as

$$Q(X, Y) - C_Y(X, Y) = -\theta(a_Y(X, Y) \cdot X + Q_Y(X, Y) \cdot Y) \quad (5.6)$$

where

$$\theta = \frac{\lambda}{1 + \lambda} \quad (5.7)$$

Also, using (3.12) (5.4) may be expressed as

$$\begin{aligned} P - C_X - c' &= -\lambda(a + a_X \cdot X + Q_X \cdot Y - C_X), \\ &= -\lambda(P + P_X x - c' + a_X X + P_Y Y - C_X). \end{aligned} \quad (5.8)$$

In the case where the budget constraint is not binding, the formula for the optimal access price takes on a very simple form.

Proposition 3

When the budget constraint is not imposed, the price of product is related to their marginal costs as  $P = C_X + c'$ . Also, the optimal access price  $a$  is expressed as  $a = P_X \cdot x + C_X$ . Hence  $a < C_X$ .

Proof) In view of (4.4) and (5.8) with  $\lambda = 0$ , we obtain

$$\begin{aligned} a &= P + P_X \cdot x - c' \\ &= (P - C_X - c') + (P_X \cdot x + C_X) \\ &= P_X x + C_X \end{aligned}$$

The formula is slightly different from that in Proposition 2. Since  $Y$  is now controlled optimally by the government, whereas in the former case  $Y$  is chosen by the monopolist who competes with rivals in the downstream markets.

The general formula (5.8) may also be written as

$$(1 + \lambda)(P - C_X - c') = -\lambda(P_X x + a_X X + P_Y Y)$$

or setting  $\theta = \lambda/(1 + \lambda)$ , using (3.5) as

$$\begin{aligned}
(P - C_X - c') &= -\theta(P_X x + a_X X + Q_X Y) \\
&= -\theta(a - P + c' + a_X X + Q_X Y).5.9
\end{aligned} \tag{5.9}$$

From this and relation (3.6) we have

$$a = -(P - C_X - c')/\theta + (P - P_Y \cdot Y) - ((n+1)P_X \cdot X + P_{XX} \cdot X^2/n) - (c' - c'' \cdot x) \tag{5.10}$$

In the benchmark case of perfect competition in the downstream industry, (5.10) reduces to

$$a = -(P - C_X - c')/\theta + (P - P_X \cdot X + P_Y \cdot Y) - (c' - c'' \cdot x) \tag{5.11}$$

We will investigate the implications of these results often some preparation.

## 6 Some Technical Lemmas

### Lemma 1

Let the utility function be of form  $W = U(X, Y) + (L_0 - L_1)$ . Given the price  $p$  of good  $X$  and the price  $q$  of good  $Y$ , let the demand functions  $X(p, q)$  and  $Y(p, q)$  be defined as the solutions of  $p = P(X, Y) = U_X(X, Y)$ ,  $q = Q(X, Y) = U_Y(X, Y)$ . Also, define the ordinary elasticities of demand by

$$\varepsilon_{XX}(p, q) = -pX_p/X, \quad \varepsilon_{XY}(p, q) = -pX_q/Y \quad \text{etc.}$$

and the inverse elasticities of demand as

$$\varepsilon_{XX}^-(X, Y) = -XP_X/P, \quad \varepsilon_{XY}^-(X, Y) = -XP_Y/Q \quad \text{etc.}$$

It then follows that

$$\begin{bmatrix} \varepsilon_{XX}^- & \varepsilon_{XY}^- \\ \varepsilon_{YX}^- & \varepsilon_{YY}^- \end{bmatrix} = \begin{bmatrix} \varepsilon_{XX} & \varepsilon_{XY} \\ \varepsilon_{YX} & \varepsilon_{YY} \end{bmatrix}^{-1}$$

and hence

$$\begin{aligned}\varepsilon_{XX}^{-} &= \varepsilon_{YY}/\Delta, & \varepsilon_{XY}^{-} &= -\varepsilon_{YX}/\Delta \\ \varepsilon_{YX}^{-} &= \varepsilon_{XY}/\Delta, & \varepsilon_{YY}^{-} &= \varepsilon_{XX}/\Delta\end{aligned}\quad (6.1)$$

where  $\Delta = \varepsilon_{XX} \cdot \varepsilon_{YY} - \varepsilon_{YX} \cdot \varepsilon_{XY}$

If  $px + qy + (L_0 - L) = M$  where  $M$  is fixed number (income) where  $l = L_0 - L$  is the leisure chosen as the numeraire and  $M$  is a fixed, we also have

$$\varepsilon_{iX} + \varepsilon_{iY} + \varepsilon_{iL} = 0 \quad (i = X, Y) \quad (6.2)$$

where  $\varepsilon_{XL} = L_p/XL$ .

Proof)

From  $P(X, Y) = p$  and  $Q(X, Y) = q$  we derive

$$\begin{pmatrix} P_X & P_Y \\ Q_X & Q_Y \end{pmatrix} \begin{pmatrix} X_P & X_Q \\ Y_P & Y_Q \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Hence we have

$$\begin{pmatrix} XP_X/P & XP_Y/Q \\ YQ_X/P & YQ_Y/Q \end{pmatrix} = \begin{pmatrix} PX_p/X & PX_q/Y \\ QY_p/X & QY_q/Y \end{pmatrix}^{-1}$$

(6.1) then follows from the definitions of ordinary and inverse elasticities.

To establish (6.2) we use the budget equation

$$pX + qY + l = M,$$

Differentiating with respect to  $p$  we have

$$pX_p + qY_p - L_p = 0.$$

Hence it follows that

$$-\frac{p}{X}X_p + \frac{q}{X}Y_p + \frac{1}{X}L_p = 0,$$

which means (6.2) for  $i = X$ . The case where  $i = Y$  is shown similarly.

## 7 Optimal Access Pricing of the Competitive Downstream Market Under the Budget Constraint

In the case where the downstream market is competitive (or contestable) , from the analysis in sections 5 and 6, we have

$$a = P(X, Y) - c'(X/n),$$

and

$$\begin{aligned}\Pi_X &= a + a_X X + Q_X Y - C_X \\ &= P + P_X \cdot X + Q_X \cdot Y - C_X - c' + c'' \cdot x,\end{aligned}$$

and

$$\begin{aligned}\Pi_Y &= a_Y + Q + Q_Y \cdot Y - C_Y \\ &= P_Y \cdot X + Q + Q_Y \cdot Y - C_Y\end{aligned}$$

The first order conditions are written as

$$P - c' - C_X = -\lambda \Pi_X \quad (7.1)$$

and

$$Q - C_Y = -\lambda \Pi_Y \quad (7.2)$$

Hence we have

$$P - c' - C_X = -\theta(P_X \cdot X + P_Y \cdot Y - c'' \cdot x) \quad (7.3)$$

$$Q - C_Y = -\theta(P_Y \cdot X + Q_Y \cdot Y). \quad (7.4)$$

Lemma 2 In the competitive case,  $\theta$  is positive if the prices are so set that the aggregate revenue of the industry exceeds the cost payments, that is if

$$R = P(X, Y) \cdot X + Q(X, Y) \cdot Y - C_X(X, Y) \cdot X - C_Y(X, Y) \cdot Y - c'(x) \cdot X$$

is positive

proof) In view of (7.3) and (7.4) we have

$$\begin{aligned} R &= (P - C_X - c')X + (Q - C_Y)Y \\ &= -\theta [P_X \cdot X + P_Y \cdot Y - c'' \cdot x]X + (P_Y \cdot X + Q_Y \cdot Y)Y \\ &= -\theta(U_{XX}X^2 + 2U_{XY} \cdot XY + U_{YY}Y^2 - nc'' \cdot x^2) \end{aligned} \quad (7.5)$$

This is positive under our assumptions.

Remark

Since  $\Pi(X, Y) - R = (a - P) \cdot X + (C - C_X X - C_Y Y) = (C - C_X X - C_Y Y)$  in the case where the downstream market is competitive, we have  $\Pi(X, Y) \geq R$  if  $C \geq C_X X - C_Y Y$ . This condition is satisfied if the technology of the monopolistic supplier of input exhibits non-decreasing returns to scale.

We now define the markup ratios of the two products by

$$m_X = (a - C_X)/P = (P - c' - C_X)/P \quad (7.6)$$

and

$$m_Y = (Q - C_Y)/Q \quad (7.7)$$

Hence we may write (7.3) and (7.4) as

$$m_X = \theta(\varepsilon_{XX}^- + \varepsilon_{YX}^-) + \theta c''(x)/P$$

and

$$m_Y = \theta(\varepsilon_{XY}^- + \varepsilon_{YY}^-).$$

We may state the another important result of our analysis. This is stated somewhat differently from the corresponding statements in Laffont and Tirole (1993, 2000) and Armstrong, Doyle and Vickers (1996).

Proposition 4

Optimal access price is higher than the direct cost of the product by the amount

$$(\theta(e_{XX}^- + e_{YX}^-) + \theta c^2(x))/P(X).$$

The proof follows from (7.6) and the expression for  $m_X$ .

In view of Lemma 1, (7.6) and (7.7) we have,

$$\begin{aligned} m_X - m_Y &= \theta(\varepsilon_{XX}^- + \varepsilon_{YX}^-) - \theta(\varepsilon_{XY}^- + \varepsilon_{YY}^-) + \theta c'' \cdot x/P \\ &= \theta(\varepsilon_{YY} - \varepsilon_{XY} + \varepsilon_{YX} - \varepsilon_{XX})/\Delta + \theta c'' \cdot x/P \\ &= \theta(\varepsilon_{XL} - \varepsilon_{YL})/\Delta + \theta c'' \cdot x/P \end{aligned}$$

Hence we can state:

Proposition 5

Optimal mark-up ratio of the industry  $X$  must be higher than that of industry  $Y$  provided that  $c''(x) \geq 0$  and  $\varepsilon_{XL} > \varepsilon_{YL}$  (if commodity  $X$  is more complementary to the numéraire good than commodity  $Y$  is). If  $c''(x) \leq 0$  and  $\varepsilon_{XL} < \varepsilon_{YL}$  the reverse conclusion holds.

We remark that a similar result has been obtained by Corlett and Hague (1988) and Diamond Mirrlees (1971) in a different context.

## 8 Effect of Entry in the Competitive Case with the Budget Constraint

We next consider the effect of entry of firms to the downstream market. Differentiating (3.2) totally with respect to  $n$ , and applying the envelope theorem, we have

$$\begin{aligned} \frac{d\mathcal{L}}{dn} &= -c(x) + c'(x)x + \lambda \Pi_n \\ &= -c(x) + c'(x)x + \lambda(c''(x) - P_X(X, Y))x^2 \end{aligned}$$

In the competitive case,  $\Pi_n = -c''(x)x^2$  and so

$$\frac{d\mathcal{L}}{dn} = -c(x) + c'(x)x + \lambda c''(x)x^2.$$



When the access price is optimal, i.e, when  $a = P - c'$ , the entrant's profit is written as

$$\begin{aligned}\pi &= (P - a)x - c(x) \\ &= c'(x)x - c(x).\end{aligned}$$

Hence if  $\pi$  is non-positive we have

$$\begin{aligned}\frac{d\mathcal{L}}{dn} &\leq -\lambda c''(x)x^2 \\ &< 0\end{aligned}$$

This establishes the following propositions

Proposition 6

Entry to the downstream industry is excessive even when the downstream industry is competitive (access price is taken as given) if the upstream industry is regulated.

In the case where the budget constraint is absent (this is if  $\lambda = 0$ ) the above result implies

$$\begin{aligned}\frac{d\mathcal{L}}{dn} &\leq -c(s) + c'(x)x \\ &= \pi.\end{aligned}$$

Hence optimal entry is assured if integer problem is neglected.

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